a) Posterior mean and 95% interval for mean and standard deviation for each school

|  |  |  |  |
| --- | --- | --- | --- |
|  | Posterior Mean | 95% Interval-Mean | 95% Interval-SD |
| School 1 | 9.300369 | (7.83145, 10.74498) | (3.007565, 5.110134) |
| School 2 | 6.96606 | (5.262186, 8.648512) | (3.367144, 5.940036) |
| School 3 | 7.869304 | (6.260536, 9.350922) | (2.782333, 5.106006) |

b) Posterior probability that θi< θj <θk for all the 6 permutation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| θ 1<2<3 | θ 1<3<2 | θ 2<1<3 | θ 2<3<1 | θ 3<1<2 | θ 3<2<1 | θ sum p |
| 0.00467 | 0.00282 | 0.07995 | 0.68493 | 0.0124 | 0.22763 | ≈1 |

c) Posterior probability that yi< yj <yk for all the 6 permutation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| y 1<2<3 | y 1<3<2 | y 2<1<3 | y 2<3<1 | y 3<1<2 | y 3<2<1 | y sum p |
| 0.10631 | 0.10199 | 0.18533 | 0.26816 | 0.14031 | 0.19777 | ≈1 |

d) Compute the probability that y1>y2 & y1>y3 , and the probability that θ1>θ2 & θ1>θ3

p(θ1>θ2 & θ1>θ3)= p(θ1>θ2 >θ3) + p(θ1>θ3 >θ2) =0.90176

p(y1>y2 & y1>y3)= p(y1>y2 >y3)+ p(y1>y3 >y2)= 0.46606

#############Code######################

######### (a) #############

S=100000

data1<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school1.dat")

data2<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school2.dat")

data3<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school3.dat")

school1 <- data1

mu.0=5;sigma.0.square=4;k.0=1;v.0=2;

n.1=length(t(school1))

k.1=k.0+n.1

mu.1=(sum(school1)+k.0\*mu.0)/k.1

v.1=v.0+n.1

sigma.1.square=1/v.1\*(v.0\*sigma.0.square+var(school1)\*(n.1-1)+(n.1\*k.0/k.1\*(mean(t(school1))-mu.0)^2))

mean.1=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))

sd.1=sqrt(1/rgamma(S,v.1/2,v.1\*sigma.1.square/2))

posterior.1<- rnorm(S,mean.1,sd.1)

mean.credible.lower=quantile(mean.1,0.025)

mean.credible.higher=quantile(mean.1,0.975)

sd.credible.lower=quantile(sd.1,0.025)

sd.credible.higher=quantile(sd.1,0.975)

mean.posterior.1=1/S\*sum(posterior.1)

print(mean.credible.lower)

print(mean.credible.higher)

print(sd.credible.lower)

print(sd.credible.higher)

print(mean.posterior.1)

################ (b) ###########

S=100000;

data1<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school1.dat")

data2<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school2.dat")

data3<-read.table("http://www.stat.washington.edu/~hoff/courses/564-2010/Data/HwData/school3.dat")

school1 <- data1

mu.0=5;sigma.0.square=4;k.0=1;v.0=2;

n.1=length(t(school1))

k.1=k.0+n.1

mu.1=(sum(school1)+k.0\*mu.0)/k.1

v.1=v.0+n.1

sigma.1.square=1/v.1\*(v.0\*sigma.0.square+var(school1)\*(n.1-1)+(n.1\*k.0/k.1\*(mean(t(school1))-mu.0)^2))

mean.1=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))

sd.1=sqrt(1/rgamma(S,v.1/2,v.1\*sigma.1.square/2))

posterior.1<- rnorm(S,mean.1,sd.1)

school1 <- data2

mu.0=5;sigma.0.square=4;k.0=1;v.0=2;

n.1=length(t(school1))

k.1=k.0+n.1

mu.1=(sum(school1)+k.0\*mu.0)/k.1

v.1=v.0+n.1

sigma.1.square=1/v.1\*(v.0\*sigma.0.square+var(school1)\*(n.1-1)+(n.1\*k.0/k.1\*(mean(t(school1))-mu.0)^2))

mean.2=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))

sd.2=sqrt(1/rgamma(S,v.1/2,v.1\*sigma.1.square/2))

posterior.2<- rnorm(S,mean.2,sd.2)

school1 <- data3

mu.0=5;sigma.0.square=4;k.0=1;v.0=2;

n.1=length(t(school1))

k.1=k.0+n.1

mu.1=(sum(school1)+k.0\*mu.0)/k.1

v.1=v.0+n.1

sigma.1.square=1/v.1\*(v.0\*sigma.0.square+var(school1)\*(n.1-1)+(n.1\*k.0/k.1\*(mean(t(school1))-mu.0)^2))

mean.3=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))

sd.3=sqrt(1/rgamma(S,v.1/2,v.1\*sigma.1.square/2))

posterior.3<- rnorm(S,mean.3,sd.3)

p.1=1/S\*sum(as.numeric((mean.1<mean.2)&(mean.2<mean.3)))

p.2=1/S\*sum(as.numeric((mean.1<mean.3)&(mean.3<mean.2)))

p.3=1/S\*sum(as.numeric((mean.2<mean.1)&(mean.1<mean.3)))

p.4=1/S\*sum(as.numeric((mean.2<mean.3)&(mean.3<mean.1)))

p.5=1/S\*sum(as.numeric((mean.3<mean.1)&(mean.1<mean.2)))

p.6=1/S\*sum(as.numeric((mean.3<mean.2)&(mean.3<mean.1)))

print(p.1);print(p.2);print(p.3);print(p.4);print(p.5);print(p.6)

################ (c) ###########

p.1=1/S\*sum(as.numeric((posterior.1<posterior.2)&(posterior.2<posterior.3)))

p.2=1/S\*sum(as.numeric((posterior.1<posterior.3)&(posterior.3<posterior.2)))

p.3=1/S\*sum(as.numeric((posterior.2<posterior.1)&(posterior.1<posterior.3)))

p.4=1/S\*sum(as.numeric((posterior.2<posterior.3)&(posterior.3<posterior.1)))

p.5=1/S\*sum(as.numeric((posterior.3<posterior.1)&(posterior.1<posterior.2)))

p.6=1/S\*sum(as.numeric((posterior.3<posterior.2)&(posterior.2<posterior.1)))

####################(d)##########

p.1=1/S\*sum(as.numeric((mean.1>mean.2)&(mean.1>mean.3)))

p.2=1/S\*sum(as.numeric((posterior.1>posterior.2)&(posterior.1>posterior.3)))